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Design of a Robust Adaptive Longitudinal Flight Control

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A recently developed adaptive controller based on a disturbance attenuation problem is applied here to the longitudinal dynamics of the F-18 high angle-of-attack research vehicle. For commanded change in angle of attack, the controller blends aerodynamic controls with thrust vectoring. The blending is based on the uncertainty of the aerodynamic control moment coefficient vs the restriction on thrust vectoring using paddles. The development of a controller for this complex control problem is based on an adaptive controller determined from solving, without any approximation, the disturbance attenuation problem, where the control coefficient matrix is uncertain. This controller estimates online not only the system state but also the uncertain parameters in the control coefficient matrix. The control is determined by maximizing a nonconcave function with respect to the uncertain parameters and is a function of the disturbance attenuation bound. In linear simulations, it is shown that as the bound is decreased, the transient response is significantly quickened, the thrust vectoring is used more heavily during the transient, and the steady state is achieved with only aerodynamic control.

I. Introduction

OVER the past several years, techniques have been developed for controlling uncertain linear systems subject to external disturbances by considering a disturbance attenuation problem. In this approach, a measure that is essentially the ratio of norms of performance outputs to disturbance inputs is created, and a robust compensator is sought that bounds this ratio below some limit. For systems with fixed parameters,[1,2] this problem was approached by converting this disturbance attenuation function to a performance index and then using a game theoretic approach to find the minimizing control for the worst-case maximizing disturbance. This approach extended the results of H_∞ analysis to include not only time-invariant systems on infinite intervals, but time-varying systems on finite intervals as well.

This disturbance attenuation approach was subsequently applied to a class of problems in which uncertainty exists in the parameters of the system control coefficient matrix.[3] Using a dynamic programming approach,[1] the problem was split into two parts. Optimizing from current to final time yielded a controller as a function of the states and parameters and an optimal return function that represents the cost to go from the current time to the final time. Optimizing from initial to current time yielded a finite-dimensional estimator structure that provides estimates of the current state and parameter values based on past measurements up to the current time, as well as an optimal accumulation function that represents the cost accumulated up to the current time. An algebraic connection condition was then determined by maximizing the sum of the optimal return function and the optimal accumulation function with respect to the states and parameters at the current time. This then resulted in a compensator structure that is both robust in that it chooses a control based on the worst-case disturbances and parameter uncertainties and adaptive in that the uncertain parameters are estimated using available measurements.

More recently,[4] it has been shown that the resulting connection condition represents a value function that satisfies the Hamilton–Jacobi–Bellman equation when all players play their saddle strategies. This is then shown to lead to an infinite time extension of the previous results. The results of Refs. 3 and 4 develop a robust adaptive controller based on a game theoretic approach without any approximation. The disturbance attenuation bound is used as a design parameter. Note that if the bound is allowed to be infinite, this approach produces the self-tuning regulator,[5] a scheme based on certainty equivalence, and gives an interpretation of that regulator in terms of this more general theory.

The problem of flight control design at high angles of attack presents a natural application of such robust and adaptive compensators. In previous work,[6] a robust controller was designed for high angle-of-attack flight conditions of the F-18 high angle-of-attack research vehicle (HARV) aircraft. The compensator was designed for zero steady-state tracking of pilot inputs by augmenting the state space with integral error states. Additionally, due to the physical limitations of the thrust vectoring hardware on the aircraft, additional washout states were added so that thrust vector commands were faded to zero in steady state. The robust compensator design was then used to expand the usable region of the linear controller about each design point effectively.

One difficulty that arises with such a design is that parameters in the linearized system may change rapidly and in some cases switch signs over dynamically varying flight conditions. By estimating the parameters that tend to vary the most and/or have the greatest effect on system performance, it may be possible to increase the overall performance of the compensator as parameter values become better known. The unique advantage of a robust adaptive compensator such as that which is presented in this paper is that by forming the control based on the worst-case values of state and parameters, the compensator can effectively use the controls whose coefficients are better known until enough measurements have been taken to reduce the uncertainty in the unknown coefficients to the point where the associated control can be used with confidence.

The longitudinal flight control for the F-18 HARV exemplifies this process. The control objective is to track a step command in angle of attack with good transient performance while fading the thrust vectoring command to zero in steady state. The essential difficulty is that during the transient period the moment coefficient due to elevator deflection M_{δ_e} is not well known. Therefore, during the transient, thrust vectoring command (TVC) δ_{TVC} should be used more than elevator deflection δ_e until the estimate of M_{δ_e} is known better with respect to a given level of uncertainty. As this occurs, the thrust vectoring can be faded out and the steady-state angle of attack is held by the elevator.

This paper shows how the theory developed in Refs. 3 and 4 can be applied to this very challenging adaptive control problem. In Sec. II, the results of Refs. 3 and 4 are reviewed, and the adaptive control algorithm is presented. Almost all of the game theoretic development and interpretation are given in Refs. 3 and 4. Only those aspects required to understand and develop the longitudinal flight control

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design are given here. In Sec. III, the design of the longitudinal flight control system using our robust adaptive control law is developed. The essential requirements are zero steady-state tracking error, good transient performance, and fading out the thrust vectoring in steady state due to restrictions on the continued use of the thrust vectoring paddles. These requirements are to be met in the presence of measurement, process, and aerodynamic uncertainty. A key feature is a design methodology that allows the thrust vectoring to be faded out in the presence of some uncertainty in the moment coefficient. This process avoids an uncontrollability condition inherent in the theory as applied to this problem.

In Sec. IV, simulation results are given in a linear simulator. The objective is to show and interpret system performance improvement by this design methodology without the complexity of introducing nonlinear dynamics and actuator dynamics. In particular, we essentially compare the self-tuning regulator, which has an infinite disturbance attenuation bound, with performance of the robust adaptive controller using various values of the disturbance attenuation bound.

II. Adaptive Control Algorithm

The problem of flight control considered is an ideal application of the theory developed in Refs. 3 and 4. The adaptive control algorithm has been developed for a linear system having uncertain coefficients that multiply one or more of the controls. The resulting control structure, outlined in this section, is both adaptive in that unknown coefficients are estimated online and robust in that the degree of uncertainty associated with the unknown coefficients is used in determining the controller gains. Thus, controls whose influences are better known are used more vigorously than those that are known with less certainty.

A. Disturbance Attenuation Problem

The problem of disturbance attenuation is one of finding a control that limits the effects of all admissible disturbances and uncertainties on the performance of the compensated system. A disturbance attenuation function is formed that is essentially a ratio of the norms of performance outputs over disturbance inputs. The problem, then, is to find a positive parameter θ such that this disturbance attenuation function is bounded. This function can be written as

$$D = \|\bar{y}\|^2/\|\bar{w}\|^2 \le 1/\theta \qquad \theta > 0 \tag{1}$$

where the measures of performance outputs and disturbance inputs are, respectively,

$$\|\bar{y}\|^2 = \|x(t_f)\|^2_{Q_f} + \int_0^{t_f} (\|x\|^2_Q + \|u\|^2_R)\,\mathrm{d}\tau \tag{2}$$

$$\|\bar{w}\|^2 = \|\xi(0)\|^2_{P_0^{-1}} + \int_0^{t_f} (\|w\|^2_{W^{-1}} + \|v\|^2_{V^{-1}})\,\mathrm{d}\tau \tag{3}$$

with x representing the states, u the controls, w the plant input disturbance, v the state measurement noise, and ξ the augmented state defined as $\xi = [x^T\ \beta^T]^T$, where β is the l-dimensional vector of unknown control coefficient matrix parameters. The dynamic system under consideration is of the form

$$\dot{x} = Ax + B(\beta)u + \Gamma w \tag{4}$$

$$z = Hx + v \tag{5}$$

where the unknown parameters β_j enter linearly into the control coefficient matrix

$$B(\beta) = B_0 + \sum_{j=1}^{l} B_j\beta_j \qquad \dot{\beta}_j = 0 \tag{6}$$

To approach this problem, we reformulate the disturbance attenuation problem as a differential game problem,[2,3] with performance index given by

$$J = \tfrac{1}{2}\{\|\bar{y}\|^2 - (1/\theta)\|\bar{w}\|^2\} \le 0 \tag{7}$$

For a given value of the disturbance attenuation bound $1/\theta$ the problem becomes one of finding the control u that minimizes this cost in the presence of the worst-case maximizing disturbance inputs provided by initial conditions $\xi(0)$ and state and measurement noise w and v.

B. Dynamic Programming Solution

The minimax problem associated with the performance index given by Eq. (7) is

$$\min_u \max_{\xi(0),w,v} J \tag{8}$$

To approach this problem, we first write the dynamics and performance index in terms of the augmented state ξ such that

$$\dot{\xi} = \bar{A}(u)\xi + \bar{B}u + \bar{\Gamma}w \tag{9}$$

$$z = \bar{H}\xi + v \tag{10}$$

where

$$\bar{A}(u) = \begin{bmatrix} A & B_1u & \cdots & B_lu \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \qquad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \qquad \bar{\Gamma} = \begin{bmatrix} \Gamma \\ 0 \end{bmatrix}$$

and

$$\bar{H} = [H \quad 0]$$

In Ref. 4, it was shown that to solve the minimax problem (8) more easily, the original partial state information problem can be converted to a full state information problem by first maximizing over initial conditions $\xi(0)$ and input disturbance w, then maximizing over measurement disturbance v. Embedding the resulting maximizing strategies within the original cost yields a one-sided full state information minimization problem, the solution of which can then be obtained using dynamic programming and has been shown in Ref. 4 to satisfy the Hamilton–Jacobi–Bellman equation.

The maximization over initial conditions and input disturbances yields an estimator structure given by

$$\dot{\hat{\xi}} = [\bar{A}(u) + \theta P\bar{Q}]\hat{\xi} + \bar{B}u + P\bar{H}^T V^{-1}(z - \bar{H}\hat{\xi}) \qquad \hat{\xi}(0) = \hat{\xi}_0 \tag{11}$$

$$\dot{P} = \bar{A}(u)P + P\bar{A}^T(u) - P(\bar{H}V^{-1}\bar{H}^T - \theta\bar{Q})P + \bar{\Gamma}W\bar{\Gamma}^T \qquad P(0) = P_0 \tag{12}$$

where

$$\bar{Q} = \begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix}$$

Embedding the maximizing initial conditions and input disturbances in the original cost and maximizing over the measurement disturbances then yields a one-sided minimization problem, the solution of which is obtained via dynamic programming.[4] The optimal return function $\bar{X}(\hat{\xi}, P, t)$ for this minimization problem is then given by

$$\bar{X}(\hat{\xi}, P, t) = \tfrac{1}{2}\max_{\xi}[x^T\Pi(\beta, t)x - (\xi - \hat{\xi})^T(\theta P)^{-1}(\xi - \hat{\xi})]$$
$$= \tfrac{1}{2}[x^{*T}\Pi(\beta^*, t)x^* - (\xi^* - \hat{\xi})^T(\theta P)^{-1}(\xi^* - \hat{\xi})] \tag{13}$$

where

$$\xi^* = \begin{bmatrix} x^* \\ \beta^* \end{bmatrix}$$

is the maximizing value of ξ in Eq. (13) and $\Pi(\beta, t)$ satisfies the Riccati equation

$$-\dot{\Pi}(\beta, t) = \Pi(\beta, t)A + A^T\Pi(\beta, t) + Q - \Pi(\beta, t)(B(\beta)R^{-1}B^T(\beta) - \theta\Gamma W\Gamma^T)\Pi(\beta, t)$$
$$\Pi(\beta, t_f) = Q_f \tag{14}$$

and the minimax control of the original problem is given by

$$u^* = -R^{-1}B^T(\beta^*)\Pi(\beta^*, t)x^* \tag{15}$$

and $\hat{\xi}$ and P are propagated by Eqs. (11) and (12), respectively.

C. Implementation

The adaptive control strategy is implemented in three steps. First, the estimator equations are propagated by Eqs. (11) and (12) as a function of past control and measurement data. Second, the maximization of the optimal value function is performed as a function of the updated values of the estimates and variance, $\hat{\beta}$ and P, to determine the worst-case state and parameter values. Third, the optimal control (15) is formed as a function of the worst-case state and the Riccati matrix $\Pi(\beta^*, t)$ obtained by evaluating Eq. (14) for the worst-case value of the unknown parameters.

The difficulty in implementing this solution is that the function being maximized in Eq. (13) is convex in the state x, but, in general, a nonconvex function of the unknown parameters β. This maximization can then be simplified somewhat by first solving the maximization with respect to the state x that gives an algebraic relation as a function of the parameter β. By the defining of

$$S \triangleq P^{-1}$$

we can partition the matrix S as

$$S = \begin{bmatrix} S_{xx} & S_{x\beta} \\ S_{\beta x} & S_{\beta\beta} \end{bmatrix}$$

The expression for the worst-case state x_t^* as a function of the parameter β is then given by

$$x^* = [\theta\Pi(\beta, t) - S_{xx}]^{-1}[S_{x\beta}(\beta - \hat{\beta}) - S_{xx}\hat{x}] \tag{16}$$

with the requirement that

$$\theta\Pi(\beta, t) - S_{xx} < 0 \qquad \forall\beta \tag{17}$$

By the substitution of Eq. (16) into Eq. (13), the maximization problem can then be solved as a function of the unknown parameters only.

Because the Riccati solution $\Pi(\beta, t)$ is dependent on β, the resulting function to be maximized becomes quite nonlinear and may have multiple local maxima. In general, with multiple uncertain parameters, the problem of finding a global maximum can be difficult to solve. However, we can make some observations that may assist in determining the global maximum. First, we notice that as the disturbance attenuation bound $1/\theta$ becomes large, the compensator essentially assumes a certainty equivalence form, such that $\beta^* = \hat{\beta}$, that is, the solution approaches that of the self-tuning regulator. With smaller values of $1/\theta$ and limited information, such that P is large, the function tends to be dominated more by the nonlinear term including $\Pi(\beta, t)$. These properties can then be utilized to assist in finding a global optimum numerically.

D. Important Properties of the Solution

It has been shown[4] that the optimal return function (13) satisfies the Hamilton–Jacobi–Bellman equation. However, it is possible that there may be cases where the global maximum is not unique. In fact, in Ref. 1 a sufficiency condition for a saddle point to exist requires that the global optimum be unique. However, these points are shown in Ref. 4 to represent a manifold of Darboux points[7] at which extremal paths loses global optimality and become merely locally optimal, which in turn create a dispersal surface[8] in the differential game problem. If all controls and disturbances play their optimal strategies, however, it has been shown[4] that the resulting extremal trajectories may originate from this manifold but they do not cross it, and therefore, the minimax problem produces a saddle point control strategy.

It is quite possible, however, that this manifold will be crossed if any of the players in the game do not play their optimal strategies. In actuality, it is very likely that the disturbances will never attain their worst-case strategies, but will more likely be random in nature. In terms of the game cost, this means that using the value of the control determined by Eq. (15) will always produce a cost that improves on the worst case. In terms of implementation, the resulting control (15) may switch from one optimizing strategy to another, based on the strategies that nature is playing, which may appear as a discontinuity as the corresponding global maximum in Eq. (13) shifts from one local maximum to another.

Additionally, it has also been shown,[4] for zero initial state and parameter estimates and the appropriate assumptions on stabilizability and detectability of the dynamic system for all possible values of the unknown parameters, that the control (15) does, in fact, satisfy the disturbance attenuation bound (1). It is important, then, to ensure that the controller does satisfy the disturbance attenuation bound, that the state space for the system to be controlled is both stabilizable and dectectable for all values of the unknown parameter β. These conditions motivate the implementation procedure in Sec. III for including the fading of thrust vectoring in the controller design.

III. Flight Controller Design

The short-period longitudinal dynamics for the F-18 HARV[9] consist of two states (angle of attack α and pitch rate q) and two controls [elevator deflection δ_e and TVC δ_{TVC}] from the dynamic model for the robust flight control design. The longitudinal dynamics are obtained from a nonlinear model by linearizing the dynamics of the airplane about a particular trim condition. For this particular example, a flight condition trimmed in steady level flight at an altitude of 25,000 ft and an angle of attack of 10 deg was selected. The basic dynamic system, then, can be written in the form

$$\dot{x}_p = A_p x_p + B_p(M_{\delta_e})u + w \tag{18}$$

$$z_p = x_p + v \tag{19}$$

with

$$x_p = \begin{bmatrix} \alpha \\ q \end{bmatrix}, \qquad u = \begin{bmatrix} \delta_e \\ \delta_{\mathrm{TVC}} \end{bmatrix}, \qquad A_p = \begin{bmatrix} Z_\alpha & Z_q \\ M_\alpha & M_q \end{bmatrix}$$

$$B_p(M_{\delta_e}) = \begin{bmatrix} Z_{\delta_e} & Z_{\delta_{\mathrm{TVC}}} \\ M_{\delta_e} & M_{\delta_{\mathrm{TVC}}} \end{bmatrix}$$

The actuator dynamics were considered to be sufficiently fast with respect to the longitudinal mode so as to be neglected.

We consider two design objectives in constructing the state space to be used in designing the compensator. The primary objective of the compensator will be to track step commands in angle of attack α, with zero steady-state error and good transient response in the presence of parameter, measurement, and process uncertainty. As a secondary objective, we would like to fade the thrust vector control command δ_{TVC} to zero in steady state to avoid damaging the paddles that are used as actuators for thrust vectoring on the F-18 HARV. Because the control effectiveness of the elevator can change at varying flight conditions, we consider M_{δ_e} as the most important uncertain parameter to be estimated online.

To accomplish our first objective, we formulate a change of variables and define the error coordinate e_α as the error between actual angle of attack α and commanded angle of attack α_c:

$$e_\alpha = \alpha - \alpha_c \tag{20}$$

To track step commands in α with zero steady-state error, a constant value of the control deflection δ_e will be required in steady state. To assure that the problem remains well posed, we must form a new state space by differentiating the error so that the control in the dynamic system used in the design synthesis is actually the derivative of the actual physical control. This assures that we have a state space for which the quadratic performance index will remain finite and that the design state and control will converge to zero as final time becomes infinite.[10] Defining an error state as

$$x_e = \begin{bmatrix} x_{e_0} \\ x_{e_1} \end{bmatrix} \qquad x_{e_0} = \begin{bmatrix} e_\alpha \\ q \end{bmatrix} \qquad x_{e_1} = \dot{x}_{e_0}$$

we have

$$\dot{\boldsymbol{x}}_e = A_e \boldsymbol{x}_e + B_e(M_{\delta_e})\dot{\boldsymbol{u}} + \Gamma_e \dot{\boldsymbol{w}} \tag{21}$$

$$z_e = H_e \boldsymbol{x}_e + \boldsymbol{v} \tag{22}$$

with

$$A_e = \begin{bmatrix} 0 & I \\ 0 & A_p \end{bmatrix}, \qquad B_e(M_{\delta_e}) = \begin{bmatrix} 0 \\ B_p(M_{\delta_e}) \end{bmatrix}$$

$$\Gamma_e = \begin{bmatrix} 0 \\ I \end{bmatrix}, \qquad H_e = [I \quad 0]$$

Next, we need to incorporate a means of fading the TVC δ_{TVC} to zero in steady state. To do this, we include δ_{TVC} in the state space, which we can then weight in our performance index so that it is driven to zero in steady state. A problem that arises when augmenting the state space in this way, however, is that the resulting state space becomes uncontrollable when the parameter M_{δ_e} goes to zero. We must then modify the state space in such a way that the system remains stabilizable for all values of M_{δ_e}. To do this, we redefine δ_e as two independent controls, $(\delta_e)_{\text{unknown}}$ and $(\delta_e)_{\text{known}}$, such that

$$\delta_e = (\delta_e)_{\text{unknown}} + (\delta_e)_{\text{known}}$$

where $(\delta_e)_{\text{known}}$ is multiplied by a fixed value of the control coefficient $(M_{\delta_e})_0$ that allows the system to remain controllable for all values of the unknown value of M_{δ_e} that multiplies $(\delta_e)_{\text{unknown}}$. Then, for small values of $(\delta_e)_{\text{known}} \approx 0$,

$$M_{\delta_e}\delta_e \approx M_{\delta_e}(\delta_e)_{\text{unknown}}$$

where $(\delta_e)_{\text{known}}$ is assured to remain small by weighting it very heavily in the performance index.

The structure of the adaptive compensator is then as shown in Fig. 1. First, the estimates of the error state $\boldsymbol{x}_e$ and the unknown parameter M_{δ_e} are formed by propagating Eqs. (11) and (12), with inputs of $\dot{\boldsymbol{u}}^*$ and z_e, where

$$\xi = \begin{bmatrix} \hat{x}_e \\ \hat{M}_{\delta_e} \end{bmatrix}, \qquad \dot{\boldsymbol{u}}^* = \begin{bmatrix} (\dot{\delta}_e)^*_{\text{unknown}} \\ (\dot{\delta}_e)^*_{\text{known}} \\ \dot{\delta}^*_{\text{TVC}} \end{bmatrix}, \qquad z_e = z_p - \begin{bmatrix} \alpha_c \\ 0 \end{bmatrix}$$

The new optimal control rates, $\dot{\boldsymbol{u}}^*$ just as defined, are then calculated as a function of ξ, P, and δ_{TVC}, so that the state used in calculating the optimal control is

$$\boldsymbol{x}_c = \begin{bmatrix} x_e \\ \delta_{\text{TVC}} \end{bmatrix}$$

In solving the maximization problem (13), we note that the state δ_{TVC} is actually something that we calculate directly and that its dynamics are decoupled from the rest of the states, so that we may simply adjoin the constraint $\delta_{\text{TVC}} = \hat{\delta}_{\text{TVC}}$ to Eq. (13). Partitioning the Riccati matrix Π in Eq. (14) as

$$\Pi = \begin{bmatrix} \Pi_{xx} & \Pi_{xu} \\ \Pi_{ux} & \Pi_{uu} \end{bmatrix}$$

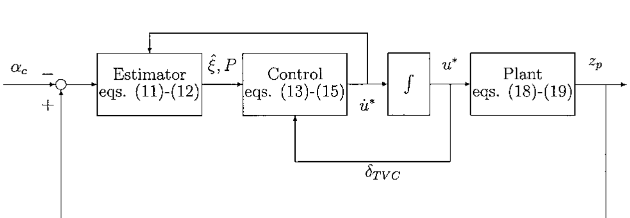

Fig. 1 Adaptive compensator block diagram.

the worst-case state as a function of the uncertain parameter β becomes

$$\boldsymbol{x}^* = [\theta\Pi_{xx}(\beta, t) - S_{xx}]^{-1}[S_{x\beta}(\beta - \hat{\beta}) - S_{xx}\hat{\boldsymbol{x}} - \theta\Pi_{xu}\delta_{\text{TVC}}] \tag{23}$$

Solving the maximization (13), $\dot{\boldsymbol{u}}^*$ is then formed as in Eq. (15). The optimal control rates are then integrated to form the actual control commands that are used in the plant.

As an additional note, for the state space and control variables available, it is not possible to both track step commands in α and fade the control δ_{TVC} to zero in steady state while simultaneously controlling the pitch rate q. For this example, then, q is not weighted in the performance index. To be able to control q we would either need to include another aerodynamic control such as flap deflection or to allow the TVC to attain a nonzero steady-state value.

IV. Simulation Results

To demonstrate the behavior of the adaptive controller at varying values of the parameter θ, step responses in angle of attack were simulated using the linearized model dynamics. A step input of 10 deg from the initial trim condition in steady level flight at 10-deg angle of attack and 25,000 ft altitude was commanded. The linearized system coefficients at this flight condition are given in Table 1. The initial state and parameter estimates were all taken to be zero to guarantee disturbance attenuation, as described in Ref. 4 for the infinite time problem.

The associated state and control weighting matrices were chosen to give acceptable step response with reasonable control deflections at the nominal values of the unknown parameter. These matrices were then chosen as

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (0.01)^2 & 0 & 0 \\ 0 & 0 & 0 & (0.02)^2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \qquad R = \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 64 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$

and the plant and measurement disturbance weightings W and V were given by

$$W = \begin{bmatrix} (0.05)^2 & 0 \\ 0 & (0.05)^2 \end{bmatrix}, \qquad V = \begin{bmatrix} (0.25)^2 & 0 \\ 0 & (0.025)^2 \end{bmatrix}$$

where the units of the input disturbances affecting angle of attack α and pitch rate q are in degrees per second and degrees per second per second, respectively, and the measurement disturbances are in degrees and degrees per second.

Simulation results are presented first for the case of worst-case input and measurement disturbances resulting from the maximization in Eq. (8) and then for an ensemble of 20 simulation runs using random disturbances with power spectral densities given by the earlier W and V. The initial true value of the unknown parameter M_{δ_e} is taken as its actual value, shown in Table 1, rather than the worst case obtained from the maximization in Eq. (8), which allows for a situation, as described in Sec. II.D, where the control strategy may switch at a given point in time. The initial state weighting P_0 was then taken to be the steady-state value of P found by assuming $\boldsymbol{u} = 0$, starting with an initial identity matrix.

Table 1 Linearized system coefficients[a]

Coefficient	Value
Z_α	−0.3367
Z_q	0.9976
M_α	−0.2065
M_q	−0.1229
Z_{δ_e}	−0.0693
$Z_{\delta_{\text{TVC}}}$	−0.0278
M_{δ_e}	−2.7320
$M_{\delta_{\text{TVC}}}$	−1.4747

[a]Here $\alpha = 10$ deg and $h = 25{,}000$ ft.

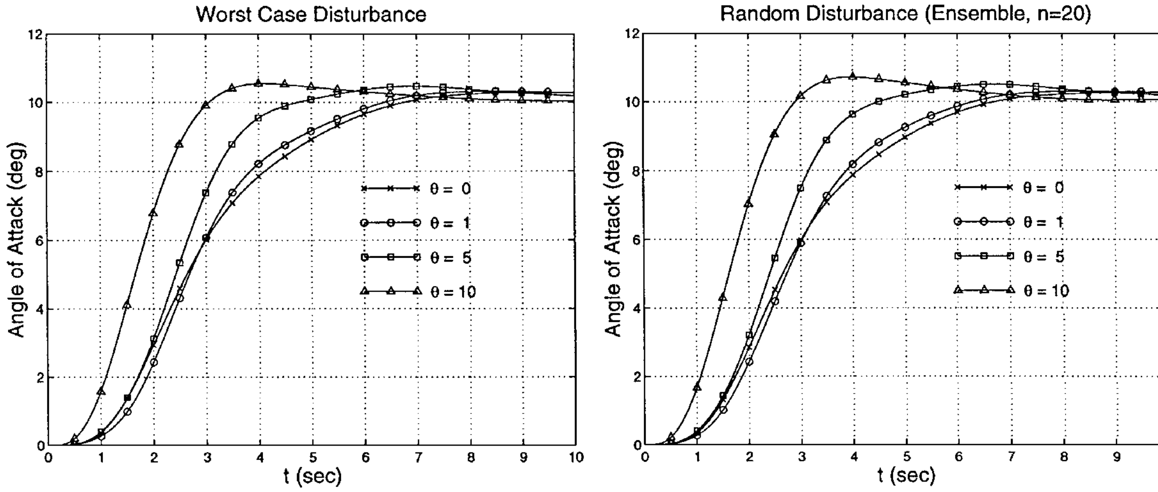

Fig. 2 Angle-of-attack step response.

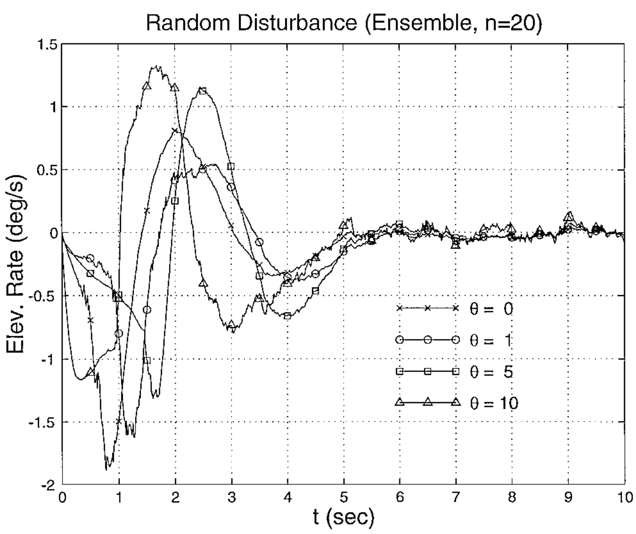

Fig. 3 Elevator command rate $\dot{\delta}_e$.

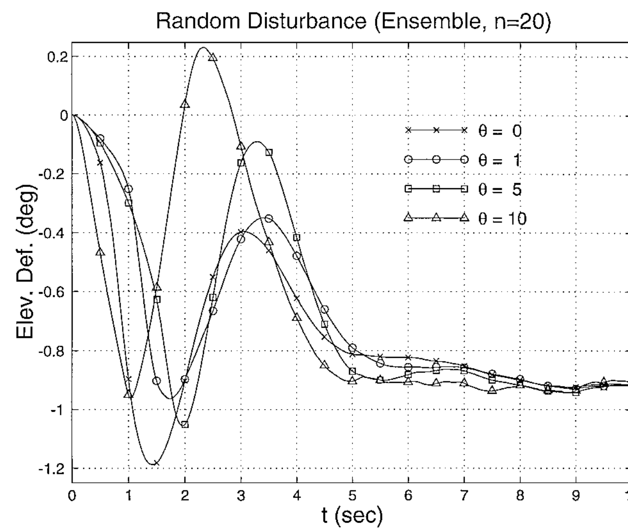

Fig. 4 Elevator command δ_e.

To determine the maximizing value of the return function (13) for $\theta > 0$, the function was first evaluated over a grid of parameter values. A local maximization was then performed originating from the maximizing parameter value along the grid. For $\theta = 0$, the return function (13) becomes a quadratic form, such that $\xi^* = \xi$, and the compensator becomes a certainty equivalence controller, equivalent to a self-tuning regulator. Simulations were performed at increasing values of θ and compared to the certainty equivalence compensator, with $\theta = 0$, to evaluate the effect of the disturbance attenuation bound on performance and robustness.

In Fig. 2, 10-deg step responses in angle of attack are presented. As θ is increased, which means that the disturbance attenuation bound is decreased, the step responses begin to exhibit slightly improved transient response. At $\theta = 5$, this improvement is somewhat more dramatic, and at $\theta = 10$, the response is much faster than at $\theta = 0$. Also, as θ increases, the average responses with random disturbance are slightly faster than those with the worst-case disturbances. Even with worst-case disturbances, though, performance does not suffer significantly from the average with random disturbance. In fact, the average responses shown in Figs. 3–10 differ only slightly from the response using worst-case disturbances.

Comparing the control commands and command rates in the average response with random disturbance, we can better understand how the increase in the parameter θ brings about the improvement in step response. For smaller values of θ, we see that the commanded elevator deflection rate $\dot{\delta}_e$, shown in Fig. 3, and elevator deflection δ_e, shown in Fig. 4, are used more heavily initially than the thrust vector command rate and thrust vector command, shown in Figs. 5 and 6. As θ is increased, however, the controller begins to hedge against the uncertainty associated with the uncertain parameter M_{δ_e} affecting the elevator command and begins to use the thrust vectoring more heavily than the elevator command. In essence, the controller hedges against using the elevator initially, due to its associated uncertainty, while relying more heavily on the thrust vectoring, whose effect is better known, until enough information is gathered to allow the elevator to be used with greater certainty.

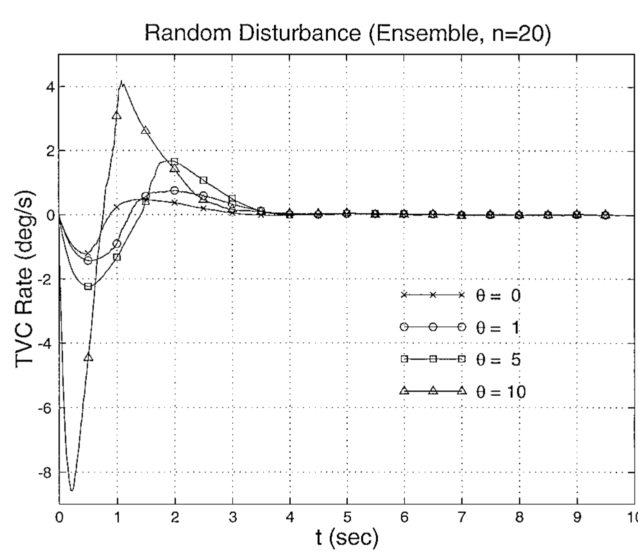

Fig. 5 Thrust vector command rate $\dot{\delta}_{TVC}$.

The estimated values of M_{δ_e} shown in Fig. 7 reflect the heavier utilization of elevator for $\theta = 0$. As θ is increased from 0, the

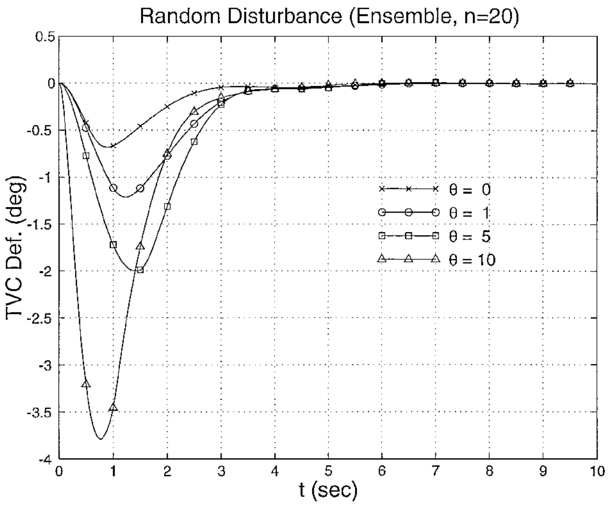

Fig. 6 Thrust vector command δ_{TVC}.

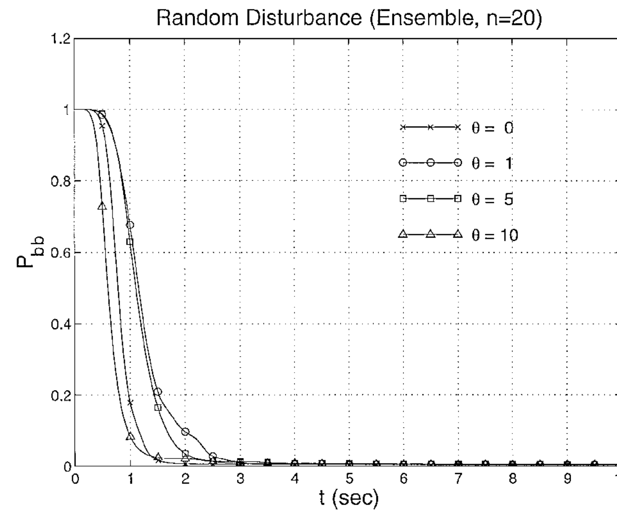

Fig. 9 Parameter variance $P_{M_{\delta_e}}$.

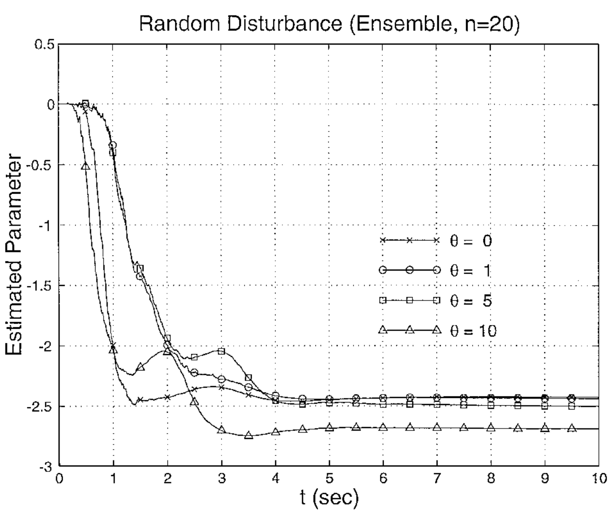

Fig. 7 Estimated parameter value $\hat{M}_{\delta_e}$, $\hat{M}_{\delta_e 0} = 2$.

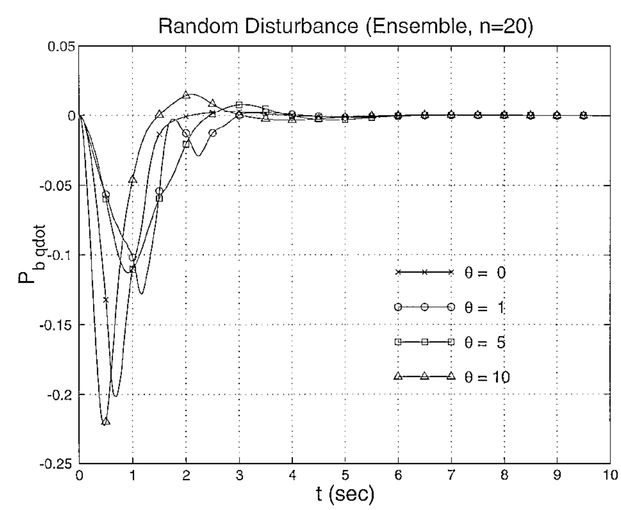

Fig. 10 Cross covariance $P_{M_{\delta_e}\dot{q}}$.

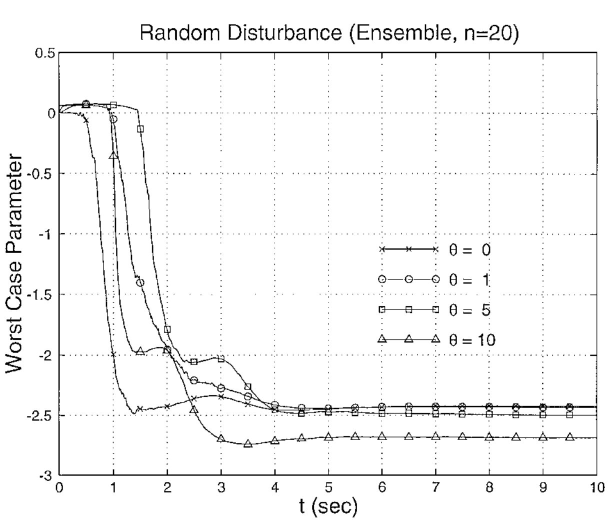

Fig. 8 Worst-case parameter $M^*_{\delta_e}$, $\hat{M}_{\delta_e 0} = 2$.

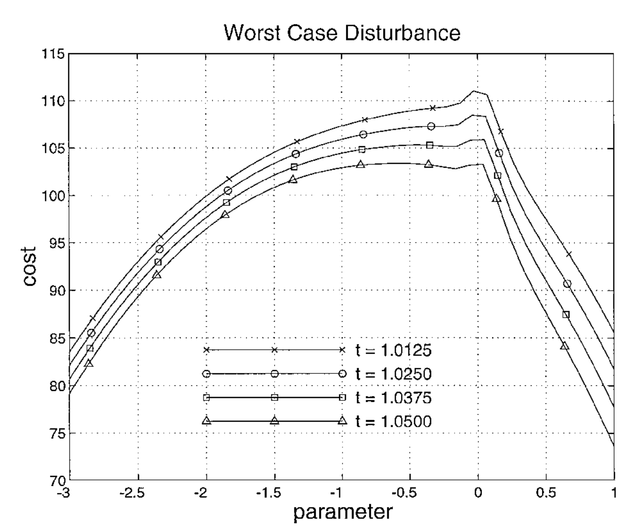

Fig. 11 Cost vs parameter, $\theta = 10$.

estimator response is initially slower, but becomes faster as θ increases, with $\theta = 10$ actually producing a faster response initially than $\theta = 0$. The parameter estimates also converge closer to the true parameter value as θ increases.

The key factor in the behavior of the controller, however, lies in the worst-case parameter value $M^*_{\delta_e}$ (Fig. 8). Initially, the true value of the parameter is highly uncertain, which is reflected in the variance of the parameter M_{δ_e}, shown in Fig. 9, and the cross covariance between the parameter and state, as shown in Fig. 10. As the parameter value becomes more well known, this worst-case value begins to follow the estimate $\hat{M}_{\delta_e}$, and the controller then begins to use the elevator deflection more confidently.

The worst-case parameter value is the value of the parameter M_{δ_e} that produces the global optimum of the cost (13). This cost is shown for increasing values of time t for the case of $\theta = 10$ with worst-case disturbance in Fig. 11. These curves demonstrate that, as time increases, two local maxima are formed. The dominant shape of the curves is that of the quadratic form centered at the estimated value of the unknown parameter, with the second smaller peak corresponding to the nonlinear portion of the cost. As time increases and more information is gathered about the unknown parameter, the peak associated with the quadratic form becomes dominant, and the controller essentially switches its strategy to use a value of M_{δ_e} closer to the estimated value. Note that this switch occurs in the control rate, which is a result of using the system of error coordinates as the design state space. In Fig. 11, this change in strategy occurs at $t = 1.05$. The end effect of this switch is that the thrust vector control, whose coefficients are known with more certainty, is used more heavily until enough information is gathered about the uncertain coefficient M_{δ_e} to use the less certain elevator deflection with confidence.

V. Conclusions

The results presented demonstrate the performance of a robust adaptive compensator based on disturbance attenuation. These results show that, as the disturbance attenuation bound is lowered, the compensator tends to rely more on controls whose effects are known with more certainty until estimates of uncertain coefficients are known with enough confidence to be used effectively.

This type of adaptive compensator shows great potential for systems having uncertain control parameters, such as those that occur in flight control. Without any a priori restrictions on the structure of the compensator, the disturbance attenuation approach results in a design that is both robust in that it tends to hedge against uncertain states and parameters and adaptive in that it uses the measurement history to update its knowledge of the unknown parameters. This type of design is particularly useful for systems, such as the flight control system examined in this paper, in which parameters may vary in magnitude and/or sign over varying conditions.

By using this robust adaptive compensator, the controller not only has the ability to update its information of the system model, but is designed in such a way that it chooses its control based on how well this model, or parameters within the model, is known. As more information becomes available to the controller, the parameters within the model become better known, and the controller is able to use this increased certainty in the system model to utilize controls that are most affected by the uncertain coefficients with more confidence. By the use of the controls that are known with the greatest certainty, the overall performance of the system can then be improved. Also, by the choice of the control that minimizes the cost based on the maximizing disturbances, the system is made robust to these disturbances, which was demonstrated in the results presented.

The main research issue that must still be addressed to further improve the implementation of this technique is the investigation of efficient means of performing the maximization in Eq. (13) by exploiting the structure of the Riccati solutions involved. The approach taken was to form a grid of parameter values over which the return function was evaluated and subsequently performing a local maximization originating from the maximizing value along the grid. This provides a reasonable method of implementation, but some tradeoff must be made between accuracy of the maximization and the computational requirements involved in evaluating a large number of grid points. Other issues being investigated include approximate techniques that may allow unknown state coefficients to be estimated as well as dynamically varying unknown parameters.

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